Sadhana Education Society

L.S. RAHEJA

COLLEGE OF ARTS & COMMERCE,

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STATISTICS II
(SYBA)

TUTORIAL WORKBOOK

PREPARED BY

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DEPARTMENT OF MATHEMATICS, STATISTICS & COMPUTERS

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Why this tutorial handbook is introduced?

"Mathematics is not about numbers, equations, computations or algorithms: it is about understanding."

_________William Paul Thurston

"Statistics is the grammar of science."

_________Karl Pearson

The Field of Statistics

The field of statistics is the science of learning from data. Statisticians offer essential insight in determining which data and conclusions are trustworthy. Statisticians know how to solve scientific mysteries and how to avoid traps that can trip up investigators.

When statistical principles are correctly applied, statistical analyses tend to produce accurate results. What's more, the analyses even account for real-world uncertainty in order to calculate the probability of being incorrect.

To produce conclusions that you can trust, statisticians must ensure that all stages of a study are correct. Statisticians know how to:

- Design studies that can answer the question at hand
- Collect trustworthy data
- Analyze data appropriately and check assumptions
- Draw reliable conclusions

It has been observed that students enrolling for F.Y.B.A lack basics of Mathematics and Statistics as some of them did not opt for Statistics in F.Y.J.C and S.Y.J.C, due to which they lose connect with mathematical concepts and rigour.

To boost the confidence of students and to make them understand Statistics lessons taught in the class and to provide them hand on practice of standard questions this tutorial handbook has been introduced.

This tutorial handbook contains:

- ✓ Latest Syllabus of Statistical techniques paper.
- ✓ Paper Pattern
- ✓ Reference Books
- ✓ Unit wise questions for practice with enough space to solve them
- ✓ Graph Papers

We hope this handbook will inculcate the problem solving aptitude among students and remove their Mathematics and Statistics phobia.

SYLLABUS FOR STATISTICS AT SYBA

Why Revision?

There is a Rapid expansion of knowledge in subject matter areas and improved instructional method during last decade. There are considerable curricular revisions happening at the high school level. Application of Mathematics and Statistics are widely used in industry and business. Keeping this in mind, a revision of syllabus required in accordance with the growth of subject of at the high school level and emerging needs of industry and its application.

Objective:

The main objective of this course is to introduce mathematics and statistics to undergraduate students of commerce, so that they can use them in the field of commerce and industry to solve the real life problems.

Distribution of topics and lectures

SEMESTER III Course Code	**Credits**
UAST 301 Statistics II	**2 Credits (45 lectures)**
Unit I: Elementary Probability Theory: Trial, random experiment, sample point and sample space. Definition of an event. Operation of events, mutually exclusive and exhaustive events. Classical (Mathematical) and Empirical definitions of Probability and their properties. Axiomatic definition of probability. Theorems on Addition and Multiplication of probabilities, pair wise.(with proof) Independence of events and mutual independence for three-events. Conditional probability, Bayes' theorem (with proof) and its applications.	**15 Lectures**
Unit II : Concept of Discrete random variable and properties of its probability distribution: Random variable. Definition and properties of probability distribution and cumulative distribution function of discrete random variable. Raw and Central moments (definition only) and their relationship. (upto order four without proof). Concepts of Skewness and Kurtosis and their uses. Expectation of a random variable. Theorems on Expectation and Variance. (with proof) Joint probability mass function of two discrete random variables. Marginal and conditional distributions. Covariance and Coefficient of Correlation. Independence of two random variables.	**15 Lectures**

Unit III : Some Standard Discrete Distributions: Discrete Uniform, Binomial and Poisson distributions and derivation of their mean and variance. Recurrence relation for probabilities of Binomial and Poisson distributions and its applications (with derivations). Poisson approximation to Binomial distribution (Statement only). Hyper geometric distribution, Derivation of its mean and variance. Binomial approximation to hyper geometric distribution (statement only)	**15 Lectures**

REFERENCES:

1. Medhi J.: Statistical Methods, An Introductory Text, Second Edition, New Age International Ltd.
2. Agarwal B.L: Basic Statistics, New Age International Ltd.
3. Spiegel M.R.: Theory and Problems of Statistics, Schaum' s Publications series.
4. Tata McGraw-Hill.
5. David S.: Elementary Probability, Cambridge University Press.
6. Hoel P.G.: Introduction to Mathematical Statistics, Asia Publishing House.
7. Hogg R.V. and Tannis E.P.: Probability and Statistical Inference. McMillan Publishing Co. Inc.
8. PitanJim: Probability, Narosa Publishing House.
9. Goon A.M., Gupta M.K., Dasgupta B.: Fundamentals of Statistics, Volume II: The World Press Private Limited, Calcutta.

Assessment of Practical Core Courses Per Semester per course:

1. Semester work, Documentation, Journal ………. 10 Marks.

2. Semester End Practical Examination ---------- 40 Marks

Semester End Examination Theory:

At the end of the semester, Theory examination of three hours duration and 100 marks based on the three units shall be held for each course. Pattern of **Theory question** paper at the end of the semester for **each course** :

There shall be Five Questions of twenty marks each.

Question 1 based on all Three units. Ten sub-questions of two marks each.

Question 2 based on Unit I (Attempt any TWO out of THREE)

Question 3 based on Unit II (Attempt any TWO out of THREE)

Question 4 based on Unit III (Attempt any TWO out of THREE)

Question 5 based on all Three Units combined. (Attempt any TWO out of THREE)

Practicals:

At the end of the semester, Practical examination of **2** hours duration and 40 marks shall be held for **each course**. Marks for term work in each paper should be given out of 10.(5 for viva and 5 for journal)

Pattern of **Practical question** paper at the end of the semester for **each course** :

There shall be Four Questions of ten marks each. Students should attempt all questions.

Question 1 based on Unit I,

Question 2 based on Unit II,

Question 3 based on Unit III,

Question 4 based on all Three Units combined.

Students should attempt **any two** sub questions out of the **three** in each Question.

Workload Theory: 3 lectures per week per course.

Practicals: 3 lecture periods per course per week per batch.

Unit I: Probability-I

Q. 1 Define the following terms with suitable examples:

1. Experiment:

2. Outcome

3. Trial

4. Random experiment

5. Sample space

6. Sample point

7. Event

8. Elementary event or simple event

9. Certain event or sure event

10. Impossible event

11. Mutually Exclusive events

12. Exhaustive events

13. Independent events

14. Complementary events

15. Sub event or subset of an event

16. Equally likely event

17. Union of two events

18. Classical definition of probability

19. Empirical definition of probability

Q. 2. Solve the following examples:

1. If a pair of unbiased coins is tossed, obtain the probability of occurrence of
    i. Single head
    ii. More than one head
    iii. At least one tail

2. A ticket is drawn from a box containing 25 tickets and a number on it is observed. Obtain the probability that ticket drawn has a number –
   i. Less than 6
   ii. Greater than 20
   iii. Multiple of 5
   iv. Lying between 10 and 15

3. One ticket is drawn at random from a box of 36 tickets numbered 1 to 36. Find the probability that the number on the ticket is either divisible by 3 or a perfect square.

4. Two dice are rolled, find the probability that the sums of the two numbers is
   i. Equal to 4
   ii. Less than 13
   iii. Greater than equal to 9
   iv. Lying between 6 and 10 both inclusive.

5. From a well shuffled pack of 52 cards, 2 cards are selected at random, what is the probability that:
   i. Both cards are red
   ii. One is red and other is black
   iii. Both are kings
   iv. Exactly one ace is drawn
   v. First is king and other is queen
   vi. Both are form the same suit

6. The letters from the word "Seminar" are arranged randomly. What is the probability that
    i. an arrangement ends with 'S'
    ii. an arrangement has all vowels occupying even places

7. A random sample of size r is selected from a population of size N with replacement, what is the probability that all elements are different?

8. A committee of 4 is to be formed from 3 engineers, 3 economists, 2 statisticians and 1 charted accountant. What is the probability that
   i. Each of the 4 categories of profession is included in the committee
   ii. The committee consists of charted accountant and at least 1 engineer.

9. Two dice are tossed. Find the probability of getting an odd number on the first die or a total of 9.

10. Events A and B are such that P(AUB)=3/4, P(AB)= 1/4 and P(A')= 2/3. Find P(B) and P(AB').

11. The probability that a student passes his first exam is 2/3 and the probability that he passes two consecutive exams is 14/45. The probability that he passes at least one examination is 4/5. What is the probability that he passes second examination?

12. There are three types of defects A, B and C. The following probabilities have been determined from available production data; P(A)=0.2, P(B)=0.16, P(C)=0.14, P(AB)=0.08, P(AC)=0.05, P(BC)=0.04, P(ABC)=0.02
    i. What is the probability that a randomly selected item of product will exhibit
    ii. At least one type of defect?
    iii. Both A and B defect but is free from type C defect?
    iv. Exactly two types of defects?

13. If 5 dice are thrown, what is the probability that first and second dice show the same face?

14. Two squares are chosen at random on chessboard of 64 squares. What is the probability that they have a side in common?

15. 12 balls are distributed at random among 3 boxes. What is the probability that the first box will contain 3 balls?

Q. 3 Theory questions.

1. What are the Limitations of classical definition of probability?

2. What are the Axioms of probability? Write down Axiomatic definition of probability.

3. State and prove Addition theorem of probability or Theorem of total probability.

4. State and prove Boole's inequality for two events.

5. Prove that the probability of an impossible event is zero and probability of certain event is 1.

Unit 2: Probability II

Q. 1. Theory Questions:

1. Explain the concept of independence of events.

2. Does mutual exclusiveness imply that events are independent?

3. Suppose A and B are independent events, define on sample space S, ten prove that:
    i. A and B' are independent
    ii. A' and B are independent
    iii. A' and B are independent

4. Explain the concept of Conditional Probability

5. Show that the conditional Probability satisfies the addition theorem.

6. State and prove Multiplication theorem for 2 events A and B.

7. State and prove Multiplication theorem for 3 events A, B and C.

8. Explain the concepts of Prior and Posterior Probability.

9. State and Prove Baye’s theorem.

Q. 2. Solve the following examples:

1. A family has 2 children. Find the probability that both children are girls if it is known that:
   i. One of the children is girl.
   ii. The older child is a girl.

2. Two fair dice are rolled. If the sum 10 has appeared, find the probability that one of the dice shows
   i. Number 4
   ii. Number 3

3. When two dice are rolled, find the probability of getting a greater number on the first die than the one on the second, given that the sum should equal 8.

4. A coin is tossed until a head appears, or until it has been tossed 3 times. Given that the head does not appear on the first toss, what is the probability that the coin is tossed 3 times.

5. Suppose a card is drawn from a well shuffled pack of cards. Let event A: getting heart card and B: getting a queen. Are A and B independent.

6. It is 8:5 against a husband who is 55 years old living till he is 75 and 4:3 against his wife who is now 48 living till she is 68. Find the probability that:
   i. The couple will be alive 20 years hence.
   ii. At least one will be alive 20years hence.

7. A husband and a wife appear for two vacancies in the same post. The probability of husband's selection is 1/6 and that of wife's selection is 1/5, what is the probability that
   i. Both will be selected
   ii. Only one of them would be selected?
   iii. None of them would be selected?

8. Lord contains 15 items of which 3 are defective stop two items are drawn at random from a lot, one after the other without replacement. Find the probability that
   i. all items are defective
   ii. first item is defective enter
   iii. only one item is defective

9. In a group of equal number of men and women, 10% main and 45% women are unemployed. What is the probability that at which person selected at random is employed?

10. Examine for paorwise and mutual independence of event K, R and S which are respectively getting of a king, red and spade card in a random draw from a well shuffled pack of 52 cards.

11. A town has three doctors A, B & C operating independently. The probability that doctor A is available is 0.9 and that for B is 0.6 and for C is 0.7; what is the probability that at least one doctor is available when needed?

12. Two types of mobiles are to be sold a salesman has 55% and 45% chances of finding customers for these two times respectively. The mobiles can be sold independently given that he was able to sell at least one of the above 2 mobiles, what is the probability that first type of mobile has been sold?

13. Considered test for Anil point the test has a known reliability. When administered to an ill person that a stool indicates with probability 0.92. When administer to a person who is not in the test will erroneously give a positive result with probability 0.04.

14. Three biased coins lie on the table. Their respective probabilities of falling heads when tossed are 1/3, 2/3, 1/4. One coin is picked up from the table using the rule, that a dice is thrown, if it shows a number 1 or 2, first coin is selected, if it shows number 3, 4 or 5, second coin is selected, otherwise third coin is selected and then a selected coin is tossed and observed to fall tail. Calculate the probability for selection of each coin.

15. A team of three students Amy, Bella and Carol answer questions in a quiz, a question is answered by Amy, or Bella or Carol with probabilities 1/2 or 1/3 or 1/6 respectively. Probability that Amy, Bella and Carol answering a question correctly are 4/5 or 3/5 or 3/5 respectively. What is the probability that a team answers a question correctly? Find the probability that Carol answered the question given that team answered incorrectly.

16. Out of 50 people surveyed in a study, 35 are smokers in which there are 20 males. What is the probability that if the person surveyed is a smoker then he is a male?

17. In a box, there are 3 red and 2 blue balls. One ball is selected at random; its colour is noted and discarded. Now second ball is selected and its colour is noted. Find the probability that both balls are blue
    i. Both balls are blue
    ii. Both balls are red
    iii. First is red and second is blue.
    iv. One is red and one is blue.

18. A bag contains 5 white 7 black balls. A ball is drawn at random, if it is white, it is removed and then second ball is drawn. IF the first ball is black, it is put back and two more black balls are added and then a second call is drawn. What is the probability that

i. Second ball drawn is white given that the first ball drawn is white
ii. Second ball drawn is black if the first ball drawn was black?
iii. Both balls are white?
iv. Both balls are of same colour?

Unit 2: Discrete Random Variable

Q. 1. Define and give example:

1. Random Variable and its types. (Discrete, Continuous)
2. Probability Mass Function
3. Cumulative Probability Distribution Function
4. Median of the distribution
5. Mode of the distribution
6. Probability distribution of function of discrete random variable
7. Mathematical expectation of a random variable.
8. Variance
9. Raw moments and Central moments
10. Joint Probability Mass Function
11. Marginal Probability Mass Function.
12. Conditional Probability Mass Function.
13. Correlation Coefficient
14. Covariance

Q. 2. Theory Questions:

1. State the properties of expectation of a discrete random variable.
2. With usual notations show that:
   i. $E(a.X + b) = a.E(x) + b$
   ii. $V(a.X + b) = a^2V(x) + b$
   iii. $Cov(aX + b, cY + d) = ac. Cov(X, Y)$
3. Show that variance is independent of change of origin but varies with change of scale.
4. Obtain the relationship between first four central moments and raw moments.
5. Explain skewness and Kurtosis with their measurements.
6. State and Prove Addition theorem of expectation of a function of two discrete random variables.
7. State and Prove Multiplication theorem of expectation of a function of two discrete random variables.
8. Define Covariance. State and prove all the properties of covariance.
9. If X and Y are two discrete random variables, then In usual notations prove that $V(aX + bY) = a^2.V(X) + b^2.V(Y) + 2ab. Cov(X, Y)$

10. State Properties of correlation coefficient.
11. Show that the range of correlation coefficient is (-1, 1).
12. Show that the magnitude of correlation coefficient is independent of change of origin and scale.
13. If X and Y are two random variables with equal variances. Show that U= X –Y and V = X +Y are uncorrelated.

Q. 3. Solve the following problems:

1. Determine the value of 'k', if P(x) is probability mass function of a random variable X.

   i. $P(x) = k \quad , x= 1, 2, 3, \ldots, 10$
      $\quad = 0 \quad$, otherwise
   ii. $P(x) = k(2x + 1) \quad , x= 0, 1, 2$
      $\quad =0 \quad$, otherwise
   iii. $P(x) = k(0.5)^x \quad , x= 1, 2, 3, \ldots$
      $\quad = 0 \quad$, otherwise
   iv. $P(x) = k(x+1)^2 \quad , x= 0, 1, 2, 3$
      $\quad = 0 \quad$, otherwise

2. If P(X) is probability mass function of a random variable X, find the value of k. Write down the expressions for it's CDF

   i.

X	:	-1	0	1	2
P(X)	:	k	2k	3k	4k

   ii.

X	:	-1	0	1	2
P(X)	:	(k+1)/13	K/13	1/13	(k-4)/13

   iii.

X	:	-1	0	1
P(X)	:	(k+2)/10	(3k+1)/10	(5-2k)/10

3. A box contains 24 bulbs of which 4 are defectives. Two bulbs are selected at random. Obtain the PMF of the number of defective bulbs in sample.
4. A box contains 4 blue and 6 red balls. Four balls are drawn at random. Obtain the PMF of the number of blue balls selected in sample.
5. The PMF of a random variable X is given by:

   i.

X	-3	-1	0	1	2	3	5	8
P(X)	0.1	0.2	0.15	0.2	0.1	0.15	0.05	0.05

Obtain its CDF. Also find $P(-1 < X \leq 2); P(1 \leq X \leq 5); P(X \leq 2)$

ii.

X	-1	0	1	2	3
P(X)	0.1	0.2	0.2	0.3	0.2

Obtain PMF of Y = X+2, W = 3X -2, V = $X^2 + 1$

iii.

X	-1	0	1	2	3
P(X)	0.1	0.2	0.2	0.3	0.2

Obtain first four raw moments hence values of mean, variance $\beta_1$and β_2.

iv.

X	4	5	6	7	8
P(X)	0.06	0.15	0..25	0.31	0.23

Obtain first four raw moments hence values of mean, variance, γ_1 and γ_2.

6. Consider an experiment of tossing a die. A die is loaded in such a way that P(X = i) = k.i, i = 1, 2, 3, 4, 5, 6
   Find the following probabilities:
   i. $P(2 < X \leq 5)$
   ii. $P(2 \leq X \leq 5)$
   iii. $P(X > 4)$
7. Consider the PMF of the random variable X.

X	1	2	3	4	5	6
P(X)	1/21	2/21	3/21	4/21	5/21	6/21

   Obtain its mean and variance.
8. For a discrete random variable X, E(X) = 10, V(X) = 25; find the positive values of a and b, such that Y = aX – b has mean 0 and variance 1.
9. A fair coin and a fair dice are thrown simultaneously, coin bears the numbers 1, 2 and numbers on dice are 1, 2, 3, 4, 5, 6. The random variable X is described as the sum of the numbers on the uppermost face of coin and dice. Write down PMF of X. Plot its graph. Also write down expression for its CDF and plot its graph.
10. A player tosses two coins. He wins ₹5 if 2 heads appear ₹2 if 1 head appears and losses ₹3 if no head appears. Find expected amount of winning and variance of amount of wining.

11. The joint PMF of (X, Y) is given below:

X / Y	1	2	3
0	0.05	0.1	0.05
1	0.1	0.2	0.15
2	0.1	0.1	0.15

Calculate E(X + Y), V(2X +3Y) and correlation coefficient between X, Y.

12. If bivariate PMF is given by,

$P_{x,\,y}(x, y) = kxy, \qquad x, y=1, 2, 3$

$\qquad\qquad = 0 \qquad\qquad$ otherwise.

Obtain the value of k. Also obtain marginal densities of X and Y. Are X and Y are independent?

13. Show that: Cov(5X + 2, 3Y – 4)

UNIT 3: Standard Discrete Distributions

Q. 1. Answer in one sentence.

1. Write down PMF of discrete Uniform Variate over the range 0, 1, 2, …., n

2. Define Bernoulli experiment.

3. State PMF of Uniform distribution over the range {1, 2, …, n}.

4. State PMF of binomial distribution.

5. State Mean and Variance of Binomial Distrubution with parameters (n, p)

6. Write down recurrence relation between probabilities of binomial variate.

7. Write down recurrence relation between probabilities of Poisson variate.

8. Write down PMF hyper-geometric variate.

9. If a random variable X follows Poisson distribution and P(X=2) = P(X=3). Obtain mean.

10. If mean of binomial distribution is 8 and variance is 6, calculate value of parameters.

11. State the conditions under which binomial distribution can be approximated to Poisson distribution.

12. State the conditions under which hypergeometric distribution can be approximated to binomial Distribution.

13. State the distribution for which
    i. Mean = variance

    ii. Mean > variance

Q. 2 Theory Questions:

1. Define Discrete Uniform variate over the range 1, 2, …, n. Write down its PMF. Obtain its mean and variance.

2. An unbiased dice is rolled. Write down the PMF for the number on uppermost face. Obtain it's mean and variance.

3. If a random variable X ~ Bin(n, p), Obtain expression for its mean and variance.

4. If a random variable X ~ Poisson (λ), Obtain expression for its mean and variance.

5. Distinguish between Binomial and Poisson Distribution.

6. Obtain PMF, expression for mean and variance for the hypergeometric distribution.

7. Write down a short note on applications of Poisson distribution.

Q. 3 Solve the following problems.

1. The probability of a man hitting the target of at a shooting range is 0. 25. If he shoots 10 times, what is the probability that he hits the target exactly 3 times? What is the probability that he hits the target at least once?

2. An unbiased coin is tossed 4 times. Calculate the probability of obtaining

   i. first no heads
   ii. at least one head
   iii. more heads and Tails.

3. For binomial variate with parameters n and p, P = q and P(X) = P(X) = 3. Find P(X) equal to one and probability of greater than one.

4. The probability of a man winning the game is 0.3. How many times must he play so that probability of winning the game at least once is at least 0.80?

5. Write down a short note on real life applications of Binomial Distribution.

6. A variate follows Poisson distribution parameters 0. 2.
   i. Find $P(X = 1)$
   ii. Find $P(X > 1)$
   iii. Find $P(X < 2)$If a random variable X follows Poisson distribution $P(X = 2) = P(X = 3)$ Find mean and $P(X=0)$.

7. In a Bolt factory bolts are active boxes with 500 words in each box. It is known that 1% bones are defective. What is the probability that one search box consists of
    i. 1 defective
    ii. More than 3 defectives.

8. Suppose we randomly select 5 cards without replacement from an ordinary Deck of playing cards. What is the probability of getting exactly two red cards?