

## SES'S L.S.RAHEJA COLLEGE OF ARTS AND COMMERCE

Course: F.Y.B.COM (MATHS)

Unit: I

Prepared by: SHRI R.B. YADAV

**INTRODUCTION:** Derivative is understood as a rate measure. When  $y$  is a function of  $x$  ( $y=f(x)$ ) i.e. quantity  $y$  depends on  $x$ , then rate of change of  $y$  w.r.t.  $x$  is called as derivative of  $y$  w.r.t.  $x$ .

and it is written as  $\frac{dy}{dx}=f'(x)$ .

Formulae on derivative:-

Sr. No	Function	Derivative	How to Read
1	Y	$\frac{dy}{dx}$	dy by dx
2	F(x)	F'(x)	f dash x
3	X <sup>n</sup>	nX <sup>n-1</sup>	n x power n-i
4	Log(x)	$\frac{1}{x}$	1 upon x
5	X	1	
6	$\frac{1}{x}$	$-\frac{1}{x^2}$	1 upon x square
7	e <sup>x</sup>	e <sup>x</sup>	e power x
8	a <sup>x</sup>	a <sup>x</sup> log(a)	a power x log(a)
9	$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	1 upon 2 root x
10	Constant (no x)	0	Zero
11	constant X function for example: 2log(x)	constant X f'(x) 2X $\frac{1}{x}$	2X derivative of log(x)

**Rules on derivatives:**

1. **Addition & Subtraction rule-**  $\frac{d}{dx}(u+v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(Derivative of Add or subtract = Addition or subtraction of derivative)

2. **Multiplication Rule**  $\frac{d}{dx}(uXv) = v \frac{du}{dx} + u \frac{dv}{dx}$

(Derivative of product = 1<sup>st</sup> x derivative of 2<sup>nd</sup> + 2<sup>nd</sup> x derivative of 1<sup>st</sup> any order)

$$v \frac{du}{dx} - u \frac{dv}{dx}$$

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3. **Division rule:**  $\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{dx \quad dx}{v^2}$

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**Solved Examples**

1. Find derivative w.r.t.  $x$   $\frac{dy}{dx}$  for the functions given below

i)  $y = (e^x + \log X)(3x^2 + 5x)$

$$\frac{dy}{dx} = (e^x + \log x)(3x^2 + 5) + (x^3 + 5x)(e^x + 1/x) \text{ Using UxV rule}$$

ii)  $Y = \frac{(3x^2 + 6x + 5)}{(2x + 3)}$

$$\frac{dy}{dx} = \frac{(2x + 3)(6x + 6) - (3x^2 + 6x + 5)(2)}{(2x + 3)^2} \text{ by } \frac{u}{v} \text{ rule}$$

iii)  $y = (2x^3 + 4x + 1)x(5x + 2)$

$$\begin{aligned} \frac{dy}{dx} &= (2x^3 + 4x + 1)x \frac{d}{dx}(5x + 2) + (5x + 2)x \frac{d}{dx}(2x^3 + 4x + 1) \quad \text{Using UxV rule} \\ &= (2x^3 + 4x + 1)x(5) + (5x + 2)x(6x^2 + 4) \end{aligned}$$

iv)  $y = \frac{(2x^2 + 5x + 6)}{(3x + 2)}$

$$\frac{dy}{dx} = \frac{(2x + 3) \frac{d}{dx}(2x^2 + 5x + 6) - (2x^2 + 5x + 6) \frac{d}{dx}(3x + 2)}{(3x + 2)^2}$$

$$\frac{dy}{dx} = \frac{(2x + 3)(4x + 5) - (2x^2 + 5x + 6)(3)}{(3x + 2)^2} \text{ by } \frac{u}{v} \text{ rule}$$

v)  $y = \frac{(x^3 + 2x + 1)}{(5x + 4)}$

$$\frac{dy}{dx} = \frac{(5x + 4) \frac{d}{dx}(x^3 + 2x + 1) - (x^3 + 2x + 1) \frac{d}{dx}(5x + 4)}{(5x + 4)^2}$$

$$\frac{dy}{dx} = \frac{(2x + 3)(3x^2 + 2) - (x^3 + 2x + 1)(5)}{(5x + 4)^2}$$

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(5x+4 \_\_\_\_\_  
)<sup>2</sup> by  
u rule —  
v

vi)  $y = (2x^2 + 5x + 4)^3$

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$$\frac{dy}{dx} = \frac{d}{dx} (2x^2 + 5x + 4)^3 \text{ (using formula } X^n) \times \frac{d}{dx} (2x^2 + 5x + 4) \text{ by Chain rule}$$

$$= 3X(2x^2 + 5x + 4)^2 \times (4x + 5)$$

$$\text{vii) } y = \frac{(4x^3 + 7x + 3)}{(5x + 1)}$$

$$\frac{dy}{dx} = \frac{(5x + 1) \frac{d}{dx} (4x^3 + 7x + 3) - (4x^3 + 7x + 3) \frac{d}{dx} (5x + 1)}{(5x + 1)^2} \text{ by } \frac{u}{v} \text{ rule}$$

$$\frac{dy}{dx} = \frac{(5x + 1)(12x^2 + 7) - (4x^3 + 7x + 3)(5)}{(5x + 1)^2}$$

$$\text{viii) } y = \frac{(3x^2 + 6x + 5)}{(2x + 3)}$$

$$\frac{dy}{dx} = \frac{(2x + 3) \frac{d}{dx} (3x^2 + 6x + 5) - (3x^2 + 6x + 5) \frac{d}{dx} (2x + 3)}{(2x + 3)^2} \text{ by } \frac{u}{v} \text{ rule}$$

$$\frac{dy}{dx} = \frac{(2x + 3)(6x + 6) - (3x^2 + 6x + 5)(2)}{(2x + 3)^2}$$

$$= \frac{(12x^2 + 30x + 18) - (6x^2 + 12x + 10)}{(2x + 3)^2} = \frac{6x^2 + 18x + 8}{(2x + 3)^2}$$

$$\text{ix) } y = (3x^2 + 5x + 2)x(4x + 1)$$

$$\frac{dy}{dx} = (3x^2 + 5x + 2) \frac{d}{dx} (4x + 1) + (4x + 1) \frac{d}{dx} (3x^2 + 5x + 2)$$

$$\frac{dy}{dx} = (3x^2 + 5x + 2) \times 4 + (4x + 1) \times (6x + 5)$$

## APPLICATIONS OF DERIVATIVE-

Meaning of derivative  $\frac{dy}{dx}$  :- When Y is a function of X (i.e. Y depends on X)  $\frac{dy}{dx}$  gives the rate of change of Y w.r.t. X.

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For example if X: Price & Y: Demand of the commodity then  $\frac{dy}{dx}$  is the rate of change in

Demand w.r.t. unit change in Price

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a) **Cost function**

**Total Cost Function (TC):** C or TC represents the Total cost of production of the commodity. It is the sum of fixed cost( $C_0$ ) & the variable cost ( $C_v$ ). Hence,  $TC=C_0+C_v$

**Average Cost (AC):** AC is the average cost of per unit production. It is calculated by dividing TC by no of units ( $x$ ). Hence,  $AC = \frac{TC}{x}$

**Marginal Cost (MC):** It is the Rate of change in the cost of production (TC) w.r.t.

production size ( $x$ ). It is obtained as the derivative of TC. Hence  $MC = \frac{d}{dx}(TC)$ .

**Marginal Average Cost (MAC):** It is the Rate of change in the average cost of production (AC) w.r.t. production size ( $x$ ). It is obtained as the derivative of AC. Hence,

$$MAC = \frac{d}{dx}(AC)$$

b) **Revenue Function:** Revenue is defined as the total sale value on the sale of quantity demanded ( $D$ ) at price ( $P$ ). It is calculated as the product of price & demand. Hence

$$TR = P \times D$$

**Average Revenue (AR):** It is the revenue generated per unit of sale. It is calculated as total revenue TR divided by demand( $D$ ) i.e. quantity sold. Hence

$$AR = \frac{TR}{D} = \frac{P \times D}{D} = P$$

In other words AR is nothing but the selling price of the commodity.

**Marginal Revenue (MR):** It is the Rate of change in the revenue w.r.t. change in demand

( $D$ ). It is obtained as the derivative of TR. Hence  $MR = \frac{dR}{dD}$

c) **Price elasticity of demand ( $\eta$ ):** Price elasticity of demand is the ratio of % change in price to % change in demand ( $D$ ). It is calculated by the formula

$$\eta = - \frac{p}{D} \frac{dD}{dp} = - \frac{\text{price}}{\text{Demand}} \times \text{Derivative of Demand}$$

d) Maximum & Minimum value of a function:

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Given any formula  $f(x)$ , to find its maximum or minimum value we follow the steps below-

- Take 1<sup>st</sup> derivative  $f'(x)$



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- Take 2<sup>nd</sup> derivative  $f''(x)$
- Solve  $f(x)=0$  ( solve 1<sup>st</sup> derivative =0) & get the value of  $x$
- Put  $x$  value in  $f''(x)$  and if
  - $f''(x)$  is negative  $f(x)$  is maximum at  $x$
  - $f''(x)$  is positive  $f(x)$  is minimum at  $x$

**SOLVED EXAMPLES:-**

1. The fixed & variable cost for a commodity are known to be Rs. 200/- and Rs.  $(5x+10)$ .

Obtain the total cost TC average cost AC for the output( $x$ ) of 25 units.

Solution: Given; fixed cost  $C_0=200$ , variable cost  $C_v= 5x+10$

Now, Total Cost of a commodity is given by  $C= C_0 + C_v$

$$Tc = 200+(5x+10)x= 5x^2 +10x+200.. \text{ Also, when } x=25$$

$$TC= 5(25)^2 +10(25)+200.= 3575$$

$$\text{And average cost } AC= \frac{TC}{x} = \frac{3575}{25} =143$$

2. The demand function of a commodity is written as  $P= D^2 +2D+15$ . Calculate total revenue TR & marginal revenue MR at known demand  $D=20$  units.

Solution: Total revenue  $TR=PxD = (D^2 +2D+15)xD= D^3 +2D^2+15D$

Also, marginal revenue

$$MR= \frac{dR}{dD} = \frac{d}{dD} (D^3 +2D^2+15D) = 3D^2 +4D+15$$

Now for demand  $D=20$ ,

$$TR= (20)^3 +2(20)^2+15(20) = 9100 \text{ and,}$$

$$MR = 3(20)^2 +4(20)+15 = 1295.$$

3. If the demand function of a commodity is  $D= 500-4xp^2$ . Find the price elasticity of demand at price  $p= 10$ /-

Solution: Price elasticity of demand:  $\eta = -\frac{p}{D} \frac{dD}{dp}$

Where, for  $p=10$   $D= 500-4x10^2=+100$  &  $\frac{dD}{dp} = \frac{d}{dp} (500-4xp^2) = -8x10=-80$

$$\text{Now, } \eta = -\frac{10}{100} x -80 = -8.$$

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4. Where, for  $p=10, D=30-2 \times 10=10$  &

$$\frac{dD}{dp} = -2 \times 10 = -20 \text{ (2Marks) Hence, } \eta = -\frac{5}{10} \times 10 = -5.$$

Remark: Price elasticity of demand ( $\eta$ ) can also be calculated from the relation,

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$$MR = AR \left(1 - \frac{1}{n}\right)$$

5. For a commodity Marginal revenue is calculated as 130. Find its price elasticity of demand at price Rs. 10/-

Solution: In this problem  $MR=130$  &  $AR=P=100$ .

We can use the relation,

$$MR = AR \left(1 - \frac{1}{n}\right) \text{ to get elasticity of demand.}$$

Putting the values we get,

$$130 = 100 \left(1 - \frac{1}{n}\right)$$

On simplification we get,

$$\text{Elasticity of demand } \eta = +\frac{2}{5}$$

6. The total cost of production for a commodity is written as  $C = (x^2 - 2x + 15)$ . Find the output (x) for the minimum cost of production.

Solution: Given,  $C = (x^2 - 2x + 15)$  to minimize the cost of production we follow the steps of minimization of  $f(x) = C = (x^2 - 2x + 15)$

Differentiating  $f(x)$  w.r.t.  $x$  we get,

$$f'(x) = \frac{d}{dx}(x^2 - 2x + 15) = 2x - 2 \text{ and,}$$

$$f''(x) = \frac{d}{dx}(2x - 2) = 2 > 0 \text{ (positive) } f(x) \text{ is min for } x, \text{ when } f'(x) = 0$$

Now when,

$$f'(x) = 0 \quad \text{i.e. } 2x - 2 = 0 \text{ gives } x = \frac{2}{2} = 1$$

Hence,  $f(x)$  is minimum for  $x = 1$  and minimum  $C$  is

$$\text{Min}(C) = 1^2 - 2 \times 1 + 15 = 1 - 2 + 15 = 14$$

7. The Total Cost of production for a commodity is given by  $C = X^2 + 20X + 25$  and its selling price is  $P = 350 - 2x$ . Find the output (x) for the maximum profit.

Solution: For the commodity,  $C = X^2 + 20X + 25$  and selling price is  $P = 350 - 2x$

Now,

$$\begin{aligned} \text{Profit} &= \text{Sales} - \text{Cost} \\ &= Px - C \\ &= (350 - 2x)X - (X^2 + 20X + 25) \end{aligned}$$

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$$= 350x - 2x^2 - X^2 - 20X - 25$$

$$= 330x - 3x^2 - 25 \text{ to be maximized. Hence, it is } f(x);$$

Therefore,

$$f(x) = 330x - 3x^2 - 25$$

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$$f'(x) = \frac{d}{dx}(-3x^2 + 330x - 25) = -6x + 330 \text{ and,}$$

$$f''(x) = \frac{d}{dx}(-6x + 330) = -6 < 0 \text{ (Negative) } f(x) \text{ is max for } x, \text{ when } f'(x) = 0$$

Now when,

$$f'(x) = 0 \quad \text{i.e. } -6x + 330 = 0 \text{ gives } x = \frac{330}{6} = 55$$

Hence,  $f(x)$  is maximum for  $x = 55$  and Max P is

$$\text{Max}(P) = -3x(55^2) - 330x55 + 25 = -9075 + 18150 - 25 = 9050.$$

## EXAMPLES FOR PRACTICE

- The demand function of a commodity is given by  $P = X^2 + 2X + 15$ , Find the AR and MR when the demand (X) is for 10 units.
  - If the fixed cost of production is 200 with variable cost Rs  $20 + x$  find the TC, AC & MC when output is of 50 units.
  - The total cost function of a commodity is given by  $C = 2x^2 + 3x + 15$ , Find the AC and MC when the output is of 10 units.
  - Calculate the price elasticity of demand for the commodity whose demand function is given by  $D = \frac{p+4}{p-2}$  at price  $p=4$ .
  - A manufacturer sells  $x$  items at price Rs  $(330-x)$ , find the output for maximum profit if his cost of production is given by  $C = x^2 + 10x + 20$ .
- Find  $\frac{dy}{dx}$ , for the following,
    - $y = (e^x + 5)(x^3 + 5x)$
    - $y = \frac{(2x^2 + 5x + 6)}{(3x + 2)}$
    - $y = \frac{(x^3 + 2x + 1)}{(5x + 4)}$
    - $y = (e^x + \log x) \cdot (2x^3 + 5x)$
    - $y = (2x^2 + 5x + 4)^3$

- The demand function of a commodity is given by  $p = 3D^3 + 5D - 1$ . Find the AR & MR when  $D = 10$  units.

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3. The demand function of a commodity is given by  $P = D^2 + 2D + 5$ , Find the AR and MR when the demand is for 10 units.
4. **Find the maximum value of the function**

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a)  $f(x) = x^2 - 4x + 7$ .    b)  $f(x) = x^3 - 9x^2 + 24x - 20$ .

5. A manufacturer sells  $x$  items at price Rs.  $(330-x)$ , find the output for maximum profit if his cost of production is given by  $C = x^2 + 10x + 20$ .
6. The total cost function of a commodity is given by  $C = 2x^2 + 3x + 15$ , Find the AC and MC when the output is of 10 units.
7. Find the elasticity of demand for the commodity whose demand function is given by

$$D = \frac{p+3}{p-1} \text{ at price } p=4.$$

Simple Interest- Interest earned on the deposit over the period of 1 year is called simple interest.

$$\text{Formula SI} = \frac{pnr}{100}$$

Where, p: Principal amount of deposit, n- period of deposit & r- rate of interest

Compound Interest- It is the interest earned on the accumulated maturity earned over the period.

$$\text{Formula CI} = P(1+r)^n - P$$

Compound interest is the interest on interest, hence  $CI > SI$  always except for first year

**Present value-** It is the amount deposited i.e. Principal or Present worth

**Future value-** It is the amount on maturity with interest.

**Annuity-** An annuity is defined as any FIXED amount paid/deposited/received on regular interval. For example installment of RD, insurance premium, loan installment or regular return on FD. This annuity is called as Annuity due where payment is received or paid at the end of the period.

$$\text{Basic formula to solve any problem, } FV = PV (1+r)^n = \frac{A(1+r)^n - 1}{r}$$

When the amount is deposited or received at the beginning of the period, it is called 'Annuity immediate'. Future value of this annuity can be calculated by adding one year interest on the Future value of Annuity due.

$$\text{Hence FV of Annuity immediate} = (PV (1+r)^n) \times (1+r)$$

## ILLUSTRATIVE EXAMPLES

1. Calculate the simple & compound interest on the sum of Rs.75000/- deposited for 4years @10% p.a.

$$\text{Formula SI} = \frac{pnr}{100} = \frac{75000 \times 4 \times 10}{100} = 30000 \text{ whereas.}$$

$$\text{Compound Int CI} = P(1+r)^n - P = 75000 \times (1+10\%)^4 - 75000 = 34807.5$$

Rs. 4807.5 extra interest earned under compound interest calculation



2. A sum of Rs.75000/- earned the interest of 15000/- over the period of 4 years what is the rate of simple interest? In how many years the sum will become double at the same rate?

**Solution:** Interest over 4 years is 15000 i.e.  $\frac{15000}{4} = 3750$  per year on Rs. 75000/-

$$\text{Rate of int} = \frac{3750}{75000} \times 100 = 5\%$$

The amount will be doubled in  $\frac{75000}{3750} = 20$  years OR **1 year 5% so 100% in 20 years.**

3. Calculate the future value of Rs.50000/- at the end of 4 years @ 10% interest compounded YLY.

Solution: This example is FV of PV-50000

Formula  $FV = PV (1+r)^n = 50000 \times (1+10\%)^4 = 73205$  is the maturity or future value.

4. What amount should be deposited in the bank so as to earn the maturity of Rs. 50000 @ 10 % compounded YLY?

Solution: This example is PV of FV 50000

Formula  $FV = PV (1+r)^n$

$$50000 = PV \times (1+10\%)^4$$

$$PV = \frac{50000}{(1+10\%)^4} = 34150 \text{ is the value of one time deposit or present value.}$$

5. Calculate the future value of regular deposit Rs.12500/- at the end of 4 years @ 10% interest compounded YLY. (Total deposit is Rs. 50000/- in 4 installment hence it is Annuity value)

Solution: This example is FV of Annuity A-12500(50000 total deposit)

$$\text{Formula } FV = \frac{A((1+r)^n - 1)}{r} = \frac{A((1+r)^n - 1)}{r} = \frac{12500 \times ((1+10\%)^4 - 1)}{10\%} = 58012.5$$

Hence amount on maturity for regular deposit of Rs. 12500 is Rs. 58012/-

6. Calculate the amount of single deposit today so as to receive regular income Rs.12500/- at the each year for 4 years @ 10% interest compounded YLY. (Total maturity Rs. 50000/- in 4 installment of 12500/- hence it is Annuity value)

Solution: This example is PV of Annuity A-12500(50000 total maturity)

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$$\text{Formula FV} = \text{PV} (1+r)^n = \frac{A(1+r)^n - 1}{r} \quad (\text{no FV})$$

On substitution we get,

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$$PV = \frac{12500x((1+10\%)^4 - 1)}{(1+10\%)^4 \times 10\%}$$

i.e.  $PV = \frac{12500x((1+10\%)^4 - 1)}{(1+10\%)^4 \times 10\%} = 39731.86$  or 39732/- is the value of single deposit.

7. What amount should be deposited at the end of each year for 4 years so as to receive one time maturity Rs. 50000/- at rate of 10% compounded YLY?

Solution: in this example one time maturity is earned by regular deposit, hence it is Annuity of Future value Rs. 50000/-. And we need to find Annuity value A

$$\text{Formula } FV = PV (1+r)^n = \frac{A(1+r)^n - 1}{r} \quad (\text{no PV})$$

On substitution we get,

$$50000 = \frac{Ax((1+10\%)^4 - 1)}{10\%}$$

$$A = \frac{50000 \times 10\%}{(1+10\%)^4 - 1} = 10867.20$$
 is the value of regular deposit to get one time maturity of

Rs. 50000/- after 4 years.

8. What amount should be deposited at the end of each year for 4 years so as to clear the loan of Rs. 50000/- at rate of 10% compounded YLY?

Solution: In this example one time payment of loan Rs. 50000.- is due today (hence it is PV) which is to be cleared repaid by regular deposit 4 installment at the end of each year hence it is Annuity value of Present value Rs. 50000/-. And we need to find Annuity value A

$$\text{Formula } FV = PV (1+r)^n = \frac{A(1+r)^n - 1}{r} \quad (\text{no FV})$$

On substitution we get,

$$50000x (1+10\%)^4 = \frac{Ax((1+10\%)^4 - 1)}{10\%}$$

$$A = \frac{50000 \times 10\% \times (1+10\%)^4}{(1+10\%)^4 - 1} = 15867.20$$
 is the value of regular deposit (installment) to

repay the loan of Rs. 50000/- in 4 years.

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Total interest paid=  $4 \times 15867 - 50000 = 13468$ .

This method of EMI calculation is also called reducing balance method.

On the other hand, EMI is also calculated by simple interest method for 4 years @ 10%.

$$\text{Then, SI} = \frac{pnr}{100} = \frac{50000 \times 4 \times 10}{100} = 20000 \text{ and EMI} = \frac{50000 + 20000}{4} = 17500.$$

This method of EMI calculation is called as Flat rate method.

**Remark:** Students can note that, in all above six calculations amount, period & rate of interest are kept same. This will make us to understand the difference in calculation in different situations. Also all the calculations can be done by using the single formula with necessary changes.

9. A sum of Rs.25000/- deposited for the period of 2years @ 8% p.a. Calculate the amount of interest earned when compounded, i) HLY ii) QLY.

Solution: HLY mode:  $r=8/2=4\%$  and  $n=2 \times 2=4$  terms

$$\text{Compound Int CI} = P(1+r)^n - P = 25000 \times (1+4\%)^4 - 25000 = 29246 - 25000 = 4246$$

Now, in QLY mode:  $r=8/4=2\%$  and  $n=2 \times 4=8$  terms

$$\text{Compound Int CI} = P(1+r)^n - P = 25000 \times (1+2\%)^8 - 25000 = 29291 - 25000 = 4291$$

Hence QLY mode earned more interest than HLY mode.

10. Calculate the effective rate of interest when 10% is compounded

- a) HLY.      b) QLY

Solution: Effective rate of interest means int earned on Rs 100 deposit for 1 year

a) HLY mode:  $r=10/2=5\%$  and  $n=1 \times 2=2$  terms

$$\text{Compound Int CI} = P(1+r)^n - P = 100 \times (1+5\%)^2 - 100 = 110.25 - 100 = 10.25\%$$

b) Now, in QLY mode:  $r=10/4=2.5\%$  and  $n=1 \times 4=4$  terms

$$\text{Compound Int CI} = P(1+r)^n - P = 100 \times (1+2.5\%)^4 - 100 = 110.38 - 100 = 10.38\%$$

Hence, in HLY mode effective rate is 10.25% whereas in QLY mode it is 10.38%

Therefore, effective rate of interest in QLY mode is more than that in HLY mode.

11. Calculate the present value of the FV Rs.75000/- at the end of 4 years @ 8% compounded YLY. (PV of FV)

$$\text{Formula FV} = PV (1+r)^n = \frac{A(1+r)^n - 1}{r} \quad (\text{no Annuity})$$

$$75000 = PV (1+8\%)^4$$

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$$PV = \frac{75000}{(1+8\%)^4} = 55127.23$$

Therefore, to get the maturity Rs. 75000 after 4 years @ 8% Rs. 55127/- has to be deposited in the bank.

12. Calculate the Future value of the Annuity value Rs. 10000/- at the end of 4 years @10% compounded YLY. (FV of A)

Solution: Formula  $FV = PV (1+r)^n = \frac{A(1+r)^n - 1}{r}$  (no PV)

Hence we have,  $FV = \frac{A((1+r)^n - 1)}{r} = \frac{10000 \times ((1+10\%)^4 - 1)}{10\%} = 464100$

Hence amount on maturity for regular deposit of Rs. 10000 is Rs. 464100/- @10% p.a.

13. What amount should be kept in the bank at the end of every year so as to get the maturity of Rs.1Lac after 4years when compounded YLY @ 12%?

Solution: Formula  $FV = PV (1+r)^n = \frac{A(1+r)^n - 1}{r}$  (no PV )

On substitution we get,

$$100000 = \frac{Ax((1+12\%)^4 - 1)}{12\%}$$

$$A = \frac{100000 \times 12\%}{(1+12\%)^4 - 1} = 20923.44 \text{ is the value of regular deposit to get one time maturity of}$$

Rs 1Lac after 4 years

14. What amount should be deposited in the bank so as to receive Rs.15000/- at the end of every year over the period of 4 years @ 14% compounded YLY.

Solution: In this example one time deposit is asked (i.e. PV) to earn regular maturity (A), Hence, it is PV of Annuity value Rs.15000/-.

Formula  $FV = PV (1+r)^n = \frac{A(1+r)^n - 1}{r}$  (no FV )

On substitution we get,

$$PV (1+14)^4 = \frac{15000 \times ((1+14\%)^4 - 1)}{14\%}$$



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4%

$$PV = \frac{15000 \times ((1+14\%)^4 - 1)}{14\% \times (1+14\%)^4} = 43705.68 = 43706 \text{ is the value of one time deposit.}$$

15. A mobile handset is purchased for Rs.45000/- by paying cash of Rs.10000/-The balance was repaid in 4 MLY installment calculated @ 36%p.a. Calculate the EMI & total interest paid.

**Solution:** In this example we can note that, one-time payment of loan Rs.35000.- (45000-1000 cash) is due today (hence it is PV) which is to be cleared repaid by regular deposit 4 MLY installments, hence it is Annuity value of Present value Rs. 35000/-. And we need to find Annuity value A=EMI

MLY rate of int  $r = 36/12 = 3\%$  &  $n = 4$  EMIs

$$\text{Formula } FV = PV (1+r)^n = \frac{A(1+r)^n - 1}{r} \quad (\text{no FV})$$

On substitution we get,

$$35000 \times (1+3\%)^4 = \frac{Ax((1+3\%)^4 - 1)}{3\%}$$

$$\text{EMI} = A = \frac{35000 \times 3\% \times (1+3\%)^4}{(1+3\%)^4 - 1} = 9415.94 = 9416 \text{ is the value of regular deposit}$$

(installment) to repay the loan of Rs.35000/- in 4 months.

$$\text{Total interest paid} = 4 \times 9416 - 35000 = 2664.$$

This method of EMI calculation is also called reducing balance method.

**SUMMERY:** For problem solving students can remember the following

- **PV-** It is the single deposit or single payment or bill payment due today
- **FV-** It is the single maturity value or bill to be cleared in future
- **A-** Annuity value is the deposit or maturity in regular installments
- **FV of PV-** it is the single maturity of single deposit. This is simple amount on maturity by compound interest.
- **PV of FV-** it is the single deposit for the single target maturity amount. Target amount collected by single deposit with compound interest.
- **A of PV-** it is the repayment of loan in regular installment. OR bill due today is cleared in regular installments of fixed amount

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- **PV of A-** it is the value of single deposit so as to get target maturity in installment (on regular basis)

- **A of FV-** It is the value of regular deposit to earn single target maturity in future. OR future provision by regular deposit. It is also called as '**Sinking Fund**'
- **FV of A-** It is the provision for future by regular deposit (saving) It is also called as '**Sinking Fund**' collected keeping in mind the future expenses in large sum. OR saving by regular deposit so as to get lump sum amount in future.

### EXAMPLES FOR PRACTICE

1. Calculate the simple interest on Rs. 25000/- @ 5% p.a. for the period of 4 years. After how many years this amount will be doubled at this rate?
2. The compound interest on a certain amount @ 8% p.a. for 2 years amounts to Rs.1250/-. Find the principal amount.
3. A sum of Rs.15000 is deposited for 2 years. What will be the amount on maturity @6% compounded half yearly (HLY)?
4. What sum will have to be deposited in the bank for 3 years so as to get Rs. 1Lac on maturity @ 8% compound interest?
5. Find the future value of the present value of Rs.12000 @ 10% p.a. for 2 years compounded half yearly.
6. Calculate the present value of RS. 1Lac at the end of 4 years compounded @ 9% p.a.
7. Shri. Rambhau deposited Rs.75000 at the end of every year for 4 years. Find the amount accumulated in his account on maturity with interest compounded @ 8%p.a.
8. Mr. Sharma wants to arrange Rs. 5Lacs for his son's admission in a Management course 4 years hence. What amt should he keep in the bank at the end of every year from now which will earn the interest @ 8% compounded annually?
9. Mr. Ashish wants to receive Rs.25000 at the end of every year for 4 years hence. Find the amount he should keep in the bank today on interest compounded @ 6%p.a.
10. Mr. Prabhat has taken a loan of Rs.50,000 from a money lender @ 24% p.a and is due to repay in 4 monthly installments. Find the EMI of the loan amount and hence calculate the

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total interest paid.

11. Mrs. Shah purchased a Washing Machine for Rs.7500 in the sale. She paid Rs. 2500 down payment & is due to pay the balance in 3monthly installments of equal amount.

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Find the EMI of the loan amount. Also calculate the total interest paid by her. Hence prepare the Amortization table.